BOUND STATE SOLUTIONS FOR CHARGED MACROSCOPIC STRINGS

Dissertation submitted by

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Date Shivangi Prasad

Certificate

This is to certify that the report on Bound state solutions for charged macroscopic strings submitted by Shivangi Prasad (07PH2004), is in partial fulfillment of the requirements for the degree of Master of Sciences, Physics. It is a bonafide record of the work done by her in the Department of Physics and Meteorology, Indian Institute of Technology, Kharagpur, under my supervision and guidance.

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Contents

Abstract

This report attempts to consolidate everything I have learnt so far about string theory. It starts with the motivation behind string theory in section 1, followed by a brief description of certain terms that one comes across. In the next section, the BPS condition for charged macroscopic strings for the case $\alpha = -\beta$ has been rederived. We go on to find the mass charge relationship, hence showing the BPS nature of the solution for the case $\alpha = \beta$. In Section 3, we have investigated the supersymmetry conditions using the Killing equation.

1 Introduction

The ultimate goal of mankind has always been to find a single unified theory, starting with the unification of electricity and magnetism to the theory of general relativity (special relativity and gravity) and further followed by the unification of special relativity and quantum mechanics(quantum field theory). The next challenge posed before the mankind is the unification of quantum mechanics and general relativity.But the standard procedures of quantization of fields do not work when applied to gravity, non-renormalizability being one of the issues in this regard.String theory stands as a strong candidate in answering this challenge.It was eventually realized that if the relativistic closed strings are quantized, massless states emerge which can be gravitons.[zweibach13]. Although, this theory cannot be experimentally tested in the near future because the present day accelerators are incapable of resolving a length of the order of $10^{-33}cm.$ size of strings, it has been successful in resolving few difficulties.
String theory assumes that the elementary particles are one dimensional extended objects-strings, instead of being point like. the different vibrational modes of these strings being identified as different particles.The dynamics of strings is governed by Nambu Goto action given by:

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d\tau\, d\sigma \sqrt{-h} \tag{1}$$

which is written analogous to the action that governs the EOM of point like particles. String theory is itself divided into several different theories-bosonic, supertring, type Ia, type Ib, type IIa, type IIb, heterotic string theory.However one theory can be transformed into other by making use of two duality transformation.

1.1 String Dualities

1) T-duality (Target Space Duality):
Maps weak coupling region of one theory to the weak coupling region of another theory.
Bosonic string theory compactified on radius 'r' and '$1/r'$ are equivalent.
Type IIa string theory compactified on a circle of radius 'r' is dual to type IIb theory compactified on a circle of radius '$1/r'$.
2) S-duality (Self Duality):

A theory with coupling constant 'g' is S-dual to some other theory with coupling constant '$1/g'$
There is third kind of duality known as U-duality which is a combination of T duality and S-duality . It relates small volume limit of some theory to large coupling limit of some other theory.

1.2 Superstring Theories

The five consistent superstring theories are:

1)The type I string has one supersymmetry in the ten-dimensional sense (16 supercharges). This theory is special in the sense that it is based on unoriented open and closed strings, while the rest are based on oriented closed strings.
2)The type II string theories have two supersymmetries in the ten-dimensional sense (32 supercharges). There are actually two kinds of type II strings called type IIA and type IIB. They differ mainly in the fact that the IIA theory is non-chiral (parity conserving) while the IIB theory is chiral (parity violating).
3)The heterotic string theories are based on a peculiar hybrid of a type I superstring and a bosonic string.

1.3 D Branes

The string end points are attached to some physical objects. These objects are what we call as D-branes, where the letter D stand for 'Dirichlet'.The name itself suggests that a set of Dirichlet boundary conditions have to be specified by the string end points. These objects are characterized by their dimensionality, more precisely, by the number of spatial dimensions that they have.In general, a Dp brane is an object with p-spatial dimensions.[5]

1.4 Charge associated with strings

As we know that a particle when couples to a Maxwell field(A_μ), carries electric charge. Similarly, strings when couple to a different kind of field, known as the Kalb-Ramond field ($B_{\mu\nu}$), carries charge. The complete action for both the system are as follows:

$$S = -m \int_P ds + \int_P A_\mu(x)\, dx^{(\mu)} - \frac{1}{k_0^2} \int d^D x\, F_{\mu\nu} F^{\mu\nu} \tag{2}$$

Similarly,

$$S = S_{str} - \frac{1}{2} \int d\tau\, d\sigma B_{\mu\nu}(X(\tau\sigma)) \frac{\partial X^{[\mu}}{\partial \tau} \frac{\partial X^{\nu]}}{\partial \sigma} - \frac{1}{6k^2} \int d^D x\, H_{\mu\nu\rho} H^{\mu\nu\rho} \tag{3}$$

where, $F_{\mu\nu}$ and $H_{\mu\nu\rho}$ are the field strengths for Maxwell field and Kalb Ramond field respectively. In both the above equations first term is the particle/string action, second term is the action due to interaction and last term is the action due to field.

1.5 D Brane Charges

A point particle, having one dimensional world line, is coupled to one index massless gauge field and a string having 2 dimensional world sheets is coupled to a two index antisymmetric, massless Kalb Ramond gauge field. Similarly, a D(p) Brane having (p+1) dimensional world volume is electrically coupled to (p+1) dimensional RR fields. However, this is true only for type IIA and type IIB closed superstring theories. In Bosonic closed string theory, the D branes are not charged. bart

$$\begin{aligned} IIA &: A_\mu(D0), A_{\mu\nu\rho}(D2) \\ IIB &: A_{\mu\nu}(D1), A_{\mu\nu\rho\sigma}(D3) \end{aligned} \tag{4}$$

2 Charged Macroscopic Strings

We start with the Charged Macroscopic string solution given in [1] with $D = 9$ and restrict ourselves to specific value of parameters α and β

2.1 $\alpha = -\beta$ solutions

The metric in ten dimensions for $\alpha = -\beta$ is given by :

$$\mathcal{G} = \begin{pmatrix} \frac{1+\frac{Ccosh^2\alpha}{r^5}}{1+\frac{C}{r^5}} & \frac{C}{r^5}\frac{cosh\alpha sinh\alpha}{(1+\frac{C}{r^5})} & 0 \\ \frac{C}{r^5}\frac{cosh\alpha sinh\alpha}{(1+\frac{C}{r^5})} & -\frac{[1-\frac{Csinh^2\alpha}{r^5}]}{(1+\frac{C}{r^5})} & 0 \\ 0 & 0 & \frac{1}{(1+\frac{C}{r^5})} \end{pmatrix} \tag{5}$$

and the antisymmetric NS-NS $B_{\mu\nu}$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & -\frac{Csinh\alpha}{(r^5+C)} \\ 0 & 0 & -\frac{Ccosh\alpha}{(r^5+C)} \\ \frac{Csinh\alpha}{(r^5+c)} & \frac{Ccosh\alpha}{(r^5+c)} & 0 \end{pmatrix} \tag{6}$$

and the dilaton is given by:

$$\phi^{(10)} = -\ln\left(1+\frac{C}{r^5}\right) \tag{7}$$

2.1.1 Generalization of D0-D2 bound states

A D-string in 10 dimensions is obtained from the above solution by application of S-Duality. The metric, antisymmetric $B_{\mu\nu}$ and dilaton is given by:

$$dS^2 = -\frac{1-\frac{C}{r^5}\sinh^2\alpha}{\sqrt{1+\frac{C}{r^5}}}(dt)^2 + \frac{1+\frac{C}{r^5}\cosh^2\alpha}{\sqrt{1+\frac{C}{r^5}}}(dx^9)^2 +$$

$$\frac{2\frac{C}{r^5}\sinh\alpha\cosh\alpha}{\sqrt{1+\frac{C}{r^5}}}dx^9 dt + \frac{1}{\sqrt{1+\frac{C}{r^5}}}(dx^8)^2 + \sqrt{1+\frac{C}{r^5}}\sum_{i=1}^{7}(dx^i)^2,$$

$$B^{(2)}_{98} = B_{98}, \qquad B^{(2)}_{t8} = B_{t8}$$

$$e^{\phi_b^{(10)}} = 1 + \frac{C}{r^5} \tag{8}$$

Next step is to apply rotation in $(x^9 - x^8)$ plane by an angle ϕ. So, we get,

$$dS^2 = -\frac{1-\frac{C}{r^5}\sinh^2\alpha}{\sqrt{1+\frac{C}{r^5}}}(dt)^2 + \frac{1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi}{\sqrt{1+\frac{C}{r^5}}}(dx^9)^2 +$$

$$\frac{1+\frac{C}{r^5}\cosh^2\alpha\sin^2\phi}{\sqrt{1+\frac{C}{r^5}}}(dx^8)^2 + \frac{2\frac{C}{r^5}\sinh\alpha\cosh\alpha\cos\phi}{\sqrt{1+\frac{C}{r^5}}}d\tilde{x}^9 dt -$$

$$\frac{2\frac{C}{r^5}\sinh\alpha\cosh\alpha\sin\phi}{\sqrt{1+\frac{C}{r^5}}}d\tilde{x}^8 dt - \frac{2\frac{C}{r^5}\cosh\alpha^2\sin\phi\cos\phi}{\sqrt{1+\frac{C}{r^5}}}d\tilde{x}^9 d\tilde{x}^8 +$$

$$\sqrt{1+\frac{C}{r^5}}\sum_{i=1}^{7}(dx^i)^2,$$

$$B^{(2)}_{\tilde{8}t} = \frac{C}{C+r^5}\cosh\alpha\cos\phi$$

$$B^{(2)}_{\tilde{9}t} = \frac{C}{C+r^5}\cosh\alpha\sin\phi$$

$$B^{(2)}_{\tilde{9}\tilde{8}} = -\frac{C}{C+r^5}\sinh\alpha \tag{9}$$

The above metric is T dualized along $\tilde{x}^9$ direction. and one obtains the following metric, NS-NS $B_{\mu\nu}$, 1-form and 3-form fields of type IIA theory.

$$dS^2 = -\frac{1+\frac{C}{r^5}(1-\cosh^2\alpha\sin^2\phi)}{\sqrt{1+\frac{C}{r^5}}(1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi)}dt^2 + \frac{\sqrt{1+\frac{C}{r^5}}}{1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi}(d\tilde{x}^9)^2 +$$
$$\frac{1+\cosh^2\alpha}{\sqrt{1+\frac{C}{r^5}}(1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi)}(d\tilde{x}^8)^2 - \frac{\frac{C}{r^5}\sinh\alpha\cosh\alpha\sin\phi}{\sqrt{1+\frac{C}{r^5}}(1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi)}dtd\tilde{x}^8$$
$$+\sqrt{1+\frac{C}{r^5}}\sum_{i=1}^{7}(dx^i)^2,$$
$$A_t = \frac{C\sin\phi\cosh\alpha}{r^5+C}, \qquad A_{\tilde{8}} = -\frac{C\sinh\alpha}{(r^5+C)},$$
$$\mathcal{B}_{\tilde{9}t} = -\frac{\frac{C}{r^5}\sinh\alpha\cosh\alpha\cos\phi}{1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi}, \qquad \mathcal{B}_{\tilde{9}\tilde{8}} = \frac{\frac{C}{r^5}\sin\phi\cos\phi\cosh^2\alpha}{1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi},$$

$$e^{\phi_b^{(10)}} = \frac{(1+\frac{C}{r^5})^(3/2)}{1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi} \qquad (10)$$

2.1.2 Generalization of D1- D3 bound state solution

We obtain a generalization of D1-D3 bound solution by T- dualizing the above generalized bound state solution along x_7.

$$
\begin{aligned}
dS^2 &= -\frac{1+\frac{C}{r^4}(1-\cosh^2\alpha\sin^2\phi)}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}dt^2 + \frac{\sqrt{1+\frac{C}{r^4}}}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}(d\tilde{x}^9)^2 + \\
&\frac{1+\cosh^2\alpha}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}(d\tilde{x}^8)^2 + \frac{1}{\sqrt{1+\frac{C}{r^4}}}(dx^7)^2 - \\
&\frac{\frac{C}{r^4}\sinh\alpha\cosh\alpha\sin\phi}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}dtd\tilde{x}^8 + \sqrt{1+\frac{C}{r^4}}\sum_{i=1}^{6}(dx^i)^2, \\
A^{(2)}_{7t} &= \frac{C\sin\phi\cosh\alpha}{r^4+C}, \qquad A^{(2)}_{7\tilde{8}} - \frac{C\sinh\alpha}{(r^4+C)}, \\
A^{(4)}_{\tilde{9}t\tilde{8}7} &= -\frac{(\frac{C}{r^4}\cosh\alpha\cos\phi)}{2(1+\frac{C}{r^4})}\left[1+\frac{1+\frac{C}{r^4}}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}\right], \\
\mathcal{B}_{\tilde{9}t} &= -\frac{\frac{C}{r^4}\sinh\alpha\cosh\alpha\cos\phi}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}, \qquad \mathcal{B}_{\tilde{9}\tilde{8}} = \frac{\frac{C}{r^4}\sin\phi\cos\phi\cosh^2\alpha}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}, \\
e^{\phi_b^{(10)}} &= \frac{1+\frac{C}{r^4}}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi} \qquad (11)
\end{aligned}
$$

2.1.3 Mass Charge Relationship

The D1 and D3 charge densities carried by the above bound state solution are given by:

$$
\begin{aligned}
Q_1 &= 4C\cosh\alpha\sin\phi, \\
Q_2 &= -4C\cosh\alpha\cos\phi, \\
Q_3 &= -4C\sinh\alpha\cosh\alpha\cos\phi, \\
P &= -4C\sinh\alpha\cosh\alpha\sin\phi \qquad (12)
\end{aligned}
$$

where, Q_1, Q_2, Q_3 and P are the charge associated with $A^{(2)}_{7t}$, $A^{(4)}_{\tilde{9}t\tilde{8}7}$, $\mathcal{B}_{\tilde{9}t}$ and $G_{t\tilde{8}}$ respectively. In order to find the ADM mass density, the deformation of the Einstein's metric above flat background.

$$\begin{aligned}
h_{77} &= \frac{C}{r^4}\left(-\frac{3}{4}+\frac{1}{4}\cosh^2\alpha\cos^2\phi\right) \\
h_{\tilde{8}\tilde{8}} &= \frac{C}{r^4}\left(\cosh^2\alpha-\frac{3}{4}\cosh^2\alpha\cos^2\phi-\frac{3}{4}\right) \\
h_{\tilde{9}\tilde{9}} &= \frac{C}{r^4}\left(\frac{1}{4}-\frac{3}{4}\cosh^2\alpha\cos^2\phi\right) \\
h_{ij} &= \frac{C}{r^4}\left(\frac{1}{4}+\frac{1}{4}\cosh^2\alpha\cos^2\phi\right)
\end{aligned} \tag{13}$$

The mass density is calculated using the following equation

$$m = \int \sum_{i=1}^{9-p} n^i \left[\sum_{j=1}^{9-p}(\partial_j h_{ij} - \partial_i h_{jj}) - \sum_{a=1}^{p}\partial_i h_{aa}\right] r^{8-p} d\Omega \tag{14}$$

which is given as

$$m_{1,3} = 4C\cosh^2\alpha \tag{15}$$

Therefore, we get

$$m_{1,3}^2 = Q_1^2 + Q_2^2 + Q_3^2 + P^2 \tag{16}$$

showing the BPS nature of the D1-D3 bound state solution.

2.2 $\alpha = \beta$ solutions

In this section, we start with the charged macroscopic string solution given in [1] and investigate the particular solution ($\alpha = \beta$). To obtain the solution in ten dimension, one has to start with the nine dimensional solution($D = 9$) and decompactify it using the Kaluza Klein compactification.[1] For this case the ten-dimensional backgrounds is given as[2]:

$$\begin{pmatrix} \hat{g} & 0 & \tilde{b} \\ 0 & -G_{tt} & 0 \\ \tilde{b} & 0 & G_{88} + \tilde{b}^2/\tilde{g} \end{pmatrix} \tag{17}$$

where, the modulus field $\tilde{g}$ is given as:

$$\tilde{g} = \frac{1 + \frac{C}{r^5}}{1 + C\cosh^2\alpha/r^5} \tag{18}$$

and $\tilde{b}$ as

$$\tilde{b} = \frac{C\sinh\alpha}{r^5 + C\cosh^2\alpha} \tag{19}$$

and the metric component G_{tt} and G_{88} as

$$G_{tt} = -\frac{1}{1 + \frac{C}{r^5}\cosh^2\alpha} \tag{20}$$

$$G_{88} = \frac{1}{1 + \frac{C}{r^5}} \tag{21}$$

The components of the gauge field are

$$\tilde{A}^1_t = 0,\ \tilde{A}^1_8 = \frac{C\sinh\alpha}{2(r^5 + C)} \tag{22}$$

$$\tilde{A}^2_t = \frac{-C\sinh\alpha\cosh\alpha}{2(r^5 + C\cosh^2\alpha)}, \tilde{A}^2_8 = 0 \tag{23}$$

and dilaton is given by the expression :

$$\phi^{(10)} = -\ln(1 + \frac{C\cosh^2\alpha}{r^5}) \tag{24}$$

The antisymmetric tensor $B_{\mu\nu}$is represented as:

$$\frac{C}{r^5 + C\cosh^2\alpha}\begin{pmatrix} 0 & -\sinh\alpha\cosh\alpha & 0 \\ \sinh\alpha\cosh\alpha & 0 & -\cosh\alpha \\ 0 & \cosh\alpha & 0 \end{pmatrix} \tag{25}$$

2.2.1 Generalization of (D0 - D2) bound states

Using the delocalized elementary string given in section 2.1, a delocalized D-string in $D = 10$ can be generated by an application of S-duality transformation The metric, antisymmetric 2-form ($B_{\mu\nu}$) and the dilaton for the delocalized D-string solution are given by:

$$
\begin{aligned}
ds^2 &= -\frac{1}{\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}}(dt)^2 + \frac{1+\frac{C}{r^5}}{\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}}(dx^9)^2 + \frac{2\frac{C}{r^5}\sinh\alpha}{\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}}dx^8dx^9 \\
&+ \frac{1+\frac{C\sinh^2\alpha}{r^5}}{\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}}(dx^8)^2 + \sqrt{1+\frac{C\cosh^2\alpha}{r^5}}\sum_{i=1}^{7}(dx^i)^2
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
B^{(2)}_{9t} &= B_{9t} = \frac{-\frac{C}{r^5}\sinh\alpha\cosh\alpha}{1+\frac{C\cosh^2\alpha}{r^5}}, \\
B^{(2)}_{8t} &= B_{8t} = \frac{-\frac{C}{r^5}\cosh\alpha}{1+\frac{C\cosh^2\alpha}{r^5}}, \\
e^{\phi_b} &= 1+\frac{C\cosh^2\alpha}{r^5}
\end{aligned}
\tag{27}
$$

The next step is rotation in $(x^8 - x^9)$ plane by an angle ϕ

$$
\begin{bmatrix} x^9 \\ x^8 \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \times \begin{bmatrix} \tilde{x}^9 \\ \tilde{x}^8 \end{bmatrix}
\tag{28}
$$

So, we get

$$
dx^9 = \tilde{dx}^9\cos\phi - \tilde{dx}^8\sin\phi \tag{29}
$$

$$
dx^8 = \tilde{dx}^9\sin\phi + \tilde{dx}^8\cos\phi \tag{30}
$$

Thus in the rotated configuration,

$$
\tilde{G}_{99} = G_{99}\cos^2\phi + G_{88}\sin^2\phi + G_{98}\cos\phi\sin\phi \tag{31}
$$

$$
\tilde{G}_{88} = G_{88}\cos^2\phi + G_{99}\sin^2\phi - G_{98}\cos\phi\sin\phi \tag{32}
$$

$$
\tilde{G}_{98} = -G_{99}\cos\phi\sin\phi + G_{88}\cos\phi\sin\phi + G_{98}\frac{\cos^2\phi - \sin^2\phi}{2} \tag{33}
$$

Thus,

$$\tilde{G}_{99} = \frac{1 + \frac{C}{r^5}\left(\cos\phi + \sinh\alpha\sin\phi\right)^2}{\sqrt{1 + \frac{C\cosh^2\alpha}{r^5}}} \tag{34}$$

$$\tilde{G}_{88} = \frac{1 + \frac{C}{r^5}\left(\sin\phi - \sinh\alpha\cos\phi\right)^2}{\sqrt{1 + \frac{C\cosh^2\alpha}{r^5}}} \tag{35}$$

$$\tilde{G}_{98} = -\frac{2\frac{C}{r^5}\left(\cos\phi + \sinh\alpha\sin\phi\right)\left(\sin\phi - \sinh\alpha\cos\phi\right)}{\sqrt{1 + \frac{C\cosh^2\alpha}{r^5}}} + \frac{\frac{C}{r^5}\sinh\alpha(\cos^2\phi - \sin^2\phi)}{\sqrt{1 + \frac{C\cosh^2\alpha}{r^5}}} \tag{36}$$

and the antisymmetric 2-form ($B_{\mu\nu}$) after rotation is given as:

$$\tilde{B}_{9t} = B_{9t}\cos\phi + B_{8t}\sin\phi = \frac{C\cosh\alpha(\sin\phi - \sinh\alpha\cos\phi)}{r^5 + C\cosh^2\alpha} \tag{37}$$

$$\tilde{B}_{8t} = -B_{9t}\sin\phi + B_{8t}\cos\phi = \frac{C\cosh\alpha(\cos\phi + \sinh\alpha\sin\phi)}{r^5 + C\cosh^2\alpha} \tag{38}$$

The rotated metric looks like this :

$$ds^2 = -\frac{1}{\sqrt{1 + \frac{C\cosh^2\alpha}{r^5}}}(dt)^2 - \frac{\frac{2C}{r^5}\left(\cos\phi + \sinh\alpha\sin\phi\right)\left(\sin\phi - \sinh\alpha\cos\phi\right)}{\sqrt{1 + \frac{C\cosh^2\alpha}{r^5}}}d\tilde{x}^8 d\tilde{x}^9 + \frac{1 + \frac{C}{r^5}\left(\cos\phi + \sinh\alpha\sin\phi\right)^2}{\sqrt{1 + \frac{C\cosh^2\alpha}{r^5}}}(d\tilde{x}^9)^2 + \frac{1 + \frac{C}{r^5}\left(\sin\phi - \sinh\alpha\cos\phi\right)^2}{\sqrt{1 + \frac{C\cosh^2\alpha}{r^5}}}(d\tilde{x}^8)^2 \sqrt{1 + \frac{C\cosh^2\alpha}{r^5}}\sum_{i=1}^{7}(dx^i)^2 \tag{39}$$

Next, we apply T-duality along $x_{\tilde{9}}$ using the prescription given in [4]:

$$G_{99}^{(a)} = \frac{1}{\tilde{G}_{99}} \tag{40}$$

$$\Rightarrow G_{99}^{(a)} = \frac{\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}}{1+\frac{C}{r^5}\left(\cos\phi+\sinh\alpha\sin\phi\right)^2}$$

$$G_{88}^{(a)} = \tilde{G}_{88} - \frac{\tilde{G}_{98}\tilde{G}_{98}}{\tilde{G}_{99}} \tag{41}$$

$$\Rightarrow G_{88}^{(a)} = \frac{\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}}{1+\frac{C}{r^5}\left(\cos\phi+\sinh\alpha\sin\phi\right)^2}$$

$$G_{00}^{(a)} = \tilde{G}_{00} + \frac{B_{9t}^2}{\tilde{G}_{99}} \tag{42}$$

$$\Rightarrow G_{00}^{(a)} = -\frac{1}{\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}}$$

The metric is :

$$ds^2 = -\frac{1}{\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}}(dt)^2 + \frac{\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}}{1+\frac{C}{r^5}\left(\cos\phi+\sinh\alpha\sin\phi\right)^2}(dx^9)^2$$
$$+\frac{\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}}{1+\frac{C}{r^5}\left(\cos\phi+\sinh\alpha\sin\phi\right)^2}(dx^8)^2$$
$$+\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}\sum_{i=1}^{7}(dx^i)^2 \tag{43}$$

$$A_t^{(a)} = \tilde{B_{9t}} \tag{44}$$

$$\Rightarrow A_t^{(a)} = \frac{C\cosh\alpha(\sin\phi-\sinh\alpha\cos\phi)}{r^5+C\cosh^2\alpha}$$

$$A^{(a)}_{9t8} = \tilde{B}_{t8} + \frac{\tilde{B}_{9t}\tilde{G}_{98}}{\tilde{G}_{99}} \tag{45}$$

$$\Rightarrow A^{(a)}_{9t8} = \frac{C\cosh\alpha(\cos\phi + \sinh\alpha\sin\phi)}{r^5 + C\cosh^2\alpha}$$

$$B^{(a)}_{98} = -\frac{\tilde{G_{98}}}{\tilde{G_{99}}} \tag{46}$$

$$\Rightarrow B^{(a)}_{98} = \frac{\frac{C}{r^5}\left(\cos\phi + \sinh\alpha\sin\phi\right)\left(\sin\phi - \sinh\alpha\cos\phi\right)}{1 + \frac{C}{r^5}\left(\cos\phi + \sinh\alpha\sin\phi\right)^2}$$

The dilaton is given as:

$$e^{2\phi_a} = \frac{e^{2\phi_b}}{\tilde{G}_{99}} \tag{47}$$

$$\Rightarrow e^{2\phi_a} = \frac{(1 + \frac{C}{r^5})^{\frac{3}{2}}}{1 + \frac{C}{r^5}\left(\cos\phi + \sinh\alpha\sin\phi\right)^2}$$

2.2.2 Generalization of (D1-D3) bound states

A generalization of $D1 - D3$ is obtained by applying T-duality along $x_7$on the generalized $D0 - D2$ solutions. The metric is given as:

$$\begin{aligned} ds^2 = &-\frac{1}{\sqrt{1 + \frac{C\cosh^2\alpha}{r^4}}}(dt)^2 + \frac{\sqrt{1 + \frac{C\cosh^2\alpha}{r^4}}}{1 + \frac{C}{r^4}\left(\cos\phi + \sinh\alpha\sin\phi\right)^2}(dx^9)^2 \\ &+\frac{\sqrt{1 + \frac{C\cosh^2\alpha}{r^4}}}{1 + \frac{C}{r^4}\left(\cos\phi + \sinh\alpha\sin\phi\right)^2}(dx^8)^2 + \frac{1}{\sqrt{1 + \frac{C\cosh^2\alpha}{r^4}}}(dx^7)^2 \\ &+\sqrt{1 + \frac{C\cosh^2\alpha}{r^4}}\sum_{i=1}^{6}(dx^i)^2 \end{aligned} \tag{48}$$

$$A_{7t} = A_t^{(a)} \tag{49}$$

$$\Rightarrow A_{7t} = \frac{\frac{C}{r^4}\cosh\alpha(\sin\phi - \sinh\alpha\cos\phi)}{1 + \frac{C}{r^4}\cosh^2\alpha}$$

$$A_{9t87} = A_{9t8} - \frac{1}{2}A_t^{(a)}B_{98}^{(a)} \tag{50}$$

$$\Rightarrow A_{9t87} = \frac{\frac{C}{r^4}\cosh\alpha(\sin\phi\sinh\alpha + \cos\phi)}{1 + \frac{C}{r^4}\cosh^2\alpha}\left(1 + \frac{\frac{C}{r^4}(\sin\phi - \sinh\alpha\cos\phi)}{1 + \frac{C}{r^4}\left(\cos\phi + \sinh\alpha\sin\phi\right)^2}\right)$$

$$B_{98} = B_{98}^{(a)} \tag{51}$$

$$\Rightarrow B_{98} = \frac{\frac{C}{r^5}\left(\cos\phi + \sinh\alpha\sin\phi\right)\left(\sin\phi - \sinh\alpha\cos\phi\right)}{1 + \frac{C}{r^5}\left(\cos\phi + \sinh\alpha\sin\phi\right)^2}$$

The dilaton is given as:

$$e^{2\phi_b} = \frac{e^{2\phi_a}}{\tilde{G}_{77}} \tag{52}$$

$$\Rightarrow e^{2\phi_b} = \frac{(1 + \frac{C}{r^4})}{1 + \frac{C}{r^4}\left(\cos\phi + \sinh\alpha\sin\phi\right)^2}$$

2.2.3 Mass Charge Relationship

The next step is to verify the BPS condition to establish the 1/2 supersymmetry of the bound state. All the non-zero charge arise from the fields that have temporal part, i.e., in the above case charges will be given by A_{7t} and A_{9t87} .

$$Q_1 = 4C\cosh\alpha(\sin\phi - \sinh\alpha\cos\phi) \tag{53}$$

$$Q_2 = 4C\cosh\alpha(\cos\phi + \sinh\alpha\sin\phi) \tag{54}$$

$$\tag{55}$$

The deformation of the Einstein metric above flat background is computed to find the ADM mass density

$$h_{77} = \frac{C}{r^4}\left(-1 - \frac{1}{2}\cosh^2\alpha + \frac{1}{4}(\cos\phi + \sin\phi\sinh\alpha)^2\right) \tag{56}$$

$$h_{88} = \frac{C}{r^4}\left(\frac{1}{2}\cosh^2\alpha - \frac{1}{4} - \frac{3}{4}(\cos\phi + \sin\phi\sinh\alpha)^2\right) \tag{57}$$

$$h_{99} = \frac{C}{r^4}\left(\frac{1}{2}\cosh^2\alpha - \frac{1}{4} - \frac{3}{4}(\cos\phi + \sin\phi\sinh\alpha)^2\right) \tag{58}$$

$$h_{ij} = \frac{C}{r^4}\left(-1 + \frac{1}{2}\cosh^2\alpha + \frac{1}{4}(\cos\phi + \sin\phi\sinh\alpha)^2\right)\delta_{ij} \tag{59}$$

The mass density of bound state is calculated using ADM mass formula given in (14)

$$m_{(1,3)} = 4C\cosh^2\alpha \tag{60}$$

We therefore have,

$$m^2_{(1,3)} = Q_1^2 + Q_2^2 \tag{61}$$

showing the BPS nature of the bound state.

3 Killing Equations

In this section we have studied killing equations for strings in ten dimensions and verified that the solutions are consistent with 1/2 supersymetry. Later, we have also shown the supersymmetry condition for D1-D3 bound states of charged macroscopic strings for the cases $\alpha = -\beta$. The spinor Killing equation for NS-NS fields, results from the supersymmetry variations in string metric [2]:

$$\delta\psi_M = \partial_M\eta + \frac{1}{4}\omega_M^{\hat{M}\hat{N}}\Gamma_{\hat{M}\hat{N}}\eta - \frac{1}{8}H_M^{\hat{M}\hat{N}}\Gamma_{\hat{M}\hat{N}}\eta^* \tag{62}$$

$$\delta\lambda = (\partial_M\phi^{(10)})\gamma^M\eta^* - \frac{1}{6}H_{MNP}\gamma^{MNP}\eta, \tag{63}$$

where ψ_M is the ten dimensional gravitino, λ is the dilatino and $\eta = (\epsilon_L + \iota\epsilon_R)$ are the supersymmetry parameters. M, N are the general coordinate indices from 0 to 9 and $\hat{M}$,$\hat{N}$ are the Lorentz indices. The first equation is separated into two equations. The indices $(9, 0, 8)$ are denoted by μ and the corresponding lorentz indices by $\hat{\mu}$. The coordinates transverse to the string (i.e. 0 to 7) are represented by m's and the lorentz indices by $\hat{m}$ The ten dimensional Lorentzian metric is of the form $\eta_{\hat{M}\hat{N}} \equiv diag(1, -1, 1, ..., 1)$ which implies, $\eta_{\hat{\mu}\hat{\nu}} = diag(1, -1, 1)$ and $\eta_{\hat{m}\hat{n}} = \delta_{\hat{m}\hat{n}}$. Since the background depends only on the transverse coordinates (here, m's) , the gravitino supersymmetry variation (3) can be written as:

$$\delta\psi_m = \partial_m\eta + \frac{1}{4}\omega_m^{\hat{\mu}\hat{\nu}}\Gamma_{\hat{\mu}\hat{\nu}}\eta - \frac{1}{8}H_m^{\hat{\mu}\hat{\nu}}\Gamma_{\hat{\mu}\hat{\nu}}\eta^*, \tag{64}$$

$$\delta\psi_\mu = \frac{1}{2}\omega_\mu^{\hat{\nu}\hat{m}}\Gamma_{\hat{\nu}\hat{m}}\eta - \frac{1}{4}H_\mu^{\hat{\nu}\hat{m}}\Gamma_{\hat{\nu}\hat{m}}\eta^* \tag{65}$$

3.1 Supersymmetry of $\beta = 0$ solution

The ten dimensional solution [2]for this case is given as :

$$ds^2 = \frac{1}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}(-dt^2 + (dx^8)^2) + \frac{\sinh^2\frac{\alpha}{2}(e^{-E} - 1)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}(dt + dx^8)^2$$
$$+\frac{\sinh\alpha(e^{-E} - 1)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}dx^9(dt + dx^8) + \sum_{i=1}^{7}(dx^i)^2 + (dx^9)^2 \tag{66}$$

The antisymmetric B is given as :

$$B_{8t} = \frac{\cosh^2\frac{\alpha}{2}(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E}-\sinh^2\frac{\alpha}{2}}, \tag{67}$$

$$B_{9t} = -\frac{\sinh\alpha}{2}\frac{(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E}-\sinh^2\frac{\alpha}{2}} = B_{98} \tag{68}$$

The dilaton is given as:

$$\phi^{(10)} = -ln\left(\cosh^2\frac{\alpha}{2}e^{-E}-\sinh^2\frac{\alpha}{2}\right) \tag{69}$$

For algebraic simplicity, the metric G and the antisymmetric tensor B are written in the form of 3×3 matrices.

$$\mathcal{G} = \begin{pmatrix} 1 & \hat{b} \\ \hat{b}^T & G+\hat{b}\hat{b}^t \end{pmatrix} \tag{70}$$

$$\mathcal{B} = \begin{pmatrix} 0 & -\hat{b} \\ \hat{b}^T & B \end{pmatrix} \tag{71}$$

In the above matrices,

$$G \equiv \begin{pmatrix} -g+a & a \\ a & g+a \end{pmatrix} \tag{72}$$

$$B \equiv \begin{pmatrix} 0 & g-1 \\ 1-g & 0 \end{pmatrix} \tag{73}$$

$$b = \frac{1}{\cosh^2\frac{\alpha}{2}e^{-E}-\sinh^2\frac{\alpha}{2}} \tag{74}$$

$$a = \frac{\sinh^2\frac{\alpha}{2}(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E}-\sinh^2\frac{\alpha}{2}} \tag{75}$$

$$g = \frac{\sinh\alpha}{2}\frac{(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E}-\sinh^2\frac{\alpha}{2}} \tag{76}$$

and $\hat{b} = b(1,1)$ is 2-dimensional row vector. The vielbien is calculated using the equation

$$\mathcal{E}\eta\mathcal{E}^{\mathcal{T}} = \mathcal{G} \tag{77}$$

The vielbien $\mathcal{E}$ is given by:

$$\mathcal{E} = \begin{pmatrix} 1 & 0 \\ \hat{b}^T & \hat{E} \end{pmatrix} \tag{78}$$

and

$$\hat{E} \equiv \frac{1}{\sqrt{g-a}} \begin{pmatrix} g-a & 0 \\ -a & g \end{pmatrix} \tag{79}$$

The spin connection matrix $\omega_m^{\hat{\mu}\hat{\nu}} = \frac{1}{2}(\mathcal{E}^T\mathcal{G}^{-1}\mathcal{E}_{,m} - \mathcal{E}^T_{,m}\mathcal{G}^{-1}\mathcal{E})$ [2] in the L.H.S of killing equation is given by:

$$\begin{pmatrix} 0 & \frac{b_{,m}}{\sqrt{g-a}} & -\frac{b_{,m}}{\sqrt{g-a}} \\ -\frac{b_{,m}}{\sqrt{g-a}} & 0 & -\frac{g_{,m}}{g} \\ \frac{b_{,m}}{\sqrt{g-a}} & \frac{g_{,m}}{g} & 0 \end{pmatrix} \tag{80}$$

And $H_m^{\hat{\mu}\hat{\nu}} = (E^T G^{-1} B_{,m} G^{-1} E)$ [2]is given by matrix

$$\begin{pmatrix} 0 & \frac{b_{,m}}{\sqrt{g-a}} & -\frac{b_{,m}}{\sqrt{g-a}} \\ -\frac{b_{,m}}{\sqrt{g-a}} & 0 & -\frac{g_{,m}}{g} + E_{,m} \\ \frac{b_{,m}}{\sqrt{g-a}} & \frac{g_{,m}}{g} - E_{,m} & 0 \end{pmatrix} \tag{81}$$

From the equation no. (3)(81) 1/2 supersymmetry condition is found as:

$$(\epsilon_L - \iota\epsilon_R) = -[\Gamma_{\hat{0}\hat{8}} + \tanh\frac{\alpha}{2} e^{E/2}(\Gamma_{\hat{9}\hat{0}} - \Gamma_{\hat{9}\hat{8}})](\epsilon_L + \iota\epsilon_R) \tag{82}$$

3.2 Supersymmetry of $\alpha = -\beta \neq 0$ solutions

The metric in ten dimensions for $\alpha = -\beta$ is given by :

$$\mathcal{G} = \begin{pmatrix} \frac{1+\frac{C cosh^2\alpha}{r^5}}{1+\frac{C}{r^5}} & \frac{C}{r^5}\frac{cosh\alpha sinh\alpha}{(1+\frac{C}{r^5})} & 0 \\ \frac{C}{r^5}\frac{cosh\alpha sinh\alpha}{(1+\frac{C}{r^5})} & -\frac{[1-\frac{C sinh^2\alpha}{r^5}]}{(1+\frac{C}{r^5})} & 0 \\ 0 & 0 & \frac{1}{(1+\frac{C}{r^5})} \end{pmatrix} \tag{83}$$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & -\frac{Csinh\alpha}{(r^5+C)} \\ 0 & 0 & -\frac{Ccosh\alpha}{(r^5+C)} \\ \frac{Csinh\alpha}{(r^5+c)} & \frac{Ccosh\alpha}{(r^5+c)} & 0 \end{pmatrix} \tag{84}$$

Again, using the same procedure we write down the metric in the following form for algebraic simplicity

$$\mathcal{G} = \begin{pmatrix} G & 0 \\ 0 & g \end{pmatrix} \tag{85}$$

In the above matrix,

$$G = \begin{pmatrix} g+k & \sqrt{kl} \\ \sqrt{kl} & -g+l \end{pmatrix} \tag{86}$$

where,

$$g = \frac{1}{1+\frac{C}{r^5}}$$

$$k = \frac{\frac{C\cosh^2\alpha}{r^5}}{1+\frac{C}{r^5}}$$

$$l = \frac{\frac{C\sinh^2\alpha}{r^5}}{1+\frac{C}{r^5}}$$

Using $\mathcal{E}\eta\mathcal{E}^{\mathcal{T}} = \mathcal{G}$, we get the vielbien as;

$$\mathcal{E} = \begin{pmatrix} \hat{E} & 0 \\ 0 & \sqrt{g} \end{pmatrix} \tag{87}$$

and $\hat{E}$ is given by;

$$\hat{E} = \begin{pmatrix} \sqrt{\frac{kl}{g-l}+g+k} & -\sqrt{\frac{kl}{g-l}} \\ 0 & \sqrt{g-l} \end{pmatrix} \tag{88}$$

Now, the matrix $H_m^{\hat{\mu}\hat{\nu}}$ is calculated using the expression given in [2]

$$H_m^{\hat{\mu}\hat{\nu}} = \frac{\partial_m[1+\frac{C}{r^5}]}{(1+\frac{C}{r^5})^2} \times \begin{pmatrix} 0 & 0 & -\frac{sinh\alpha}{G_{88}\sqrt{G_{tt}}} \\ 0 & 0 & \frac{cosh\alpha}{\sqrt{G_{tt}G_{88}}} \\ \frac{sinh\alpha}{G_{88}\sqrt{G_{tt}}} & -\frac{cosh\alpha}{\sqrt{G_{tt}G_{88}}} & 0 \end{pmatrix} \tag{89}$$

From the dilatino variation equation, we arrive at the 1/2 supersymmetry condition.

$$(\epsilon_L - \iota\epsilon_R) = \left[-\frac{\cosh\alpha}{\sqrt{1 - \frac{C\sinh^2\alpha}{r^5}}}\Gamma_{\hat{0}\hat{8}} + \sinh\alpha\sqrt{\frac{1+\frac{C}{r^5}}{1-\frac{C\sinh^2\alpha}{r^5}}}\Gamma_{\hat{9}\hat{8}} \right](\epsilon_L + \iota\epsilon_R) \quad (90)$$

3.3 Supersymmetry conditions for D1-D3 bound state solution for charged macroscopic strings for $\alpha = -\beta$

The following is the D1-D3 bound state solution for the case $\alpha = -\beta$

$$dS^2 = -\frac{1+\frac{C}{r^4}(1-\cosh^2\alpha\sin^2\phi)}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}dt^2 + \frac{\sqrt{1+\frac{C}{r^4}}}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}(d\tilde{x}^9)^2 +$$
$$\frac{1+\cosh^2\alpha}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}(d\tilde{x}^8)^2 + \frac{1}{\sqrt{1+\frac{C}{r^4}}}(dx^7)^2 -$$
$$\frac{\frac{C}{r^4}\sinh\alpha\cosh\alpha\sin\phi}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}dtd\tilde{x}^8 + \sqrt{1+\frac{C}{r^4}}\sum_{i=1}^{6}(dx^i)^2,$$
$$A^{(2)}_{7t} = \frac{C\sin\phi\cosh\alpha}{r^4+C}, \qquad A^{(2)}_{7\tilde{8}} - \frac{C\sinh\alpha}{(r^4+C)},$$
$$A^{(4)}_{\tilde{9}t\tilde{8}7} = -\frac{(\frac{C}{r^4}\cosh\alpha\cos\phi)}{2(1+\frac{C}{r^4})}\left[1+\frac{1+\frac{C}{r^4}}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}\right],$$
$$\mathcal{B}_{\tilde{9}t} = -\frac{\frac{C}{r^4}\sinh\alpha\cosh\alpha\cos\phi}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}, \qquad \mathcal{B}_{\tilde{9}\tilde{8}} = \frac{\frac{C}{r^4}\sin\phi\cos\phi\cosh^2\alpha}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi},$$
$$e^{\phi_b^{(10)}} = \frac{1+\frac{C}{r^4}}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi} \qquad (91)$$

$$\mathcal{G} = \begin{pmatrix} g & 0 \\ 0 & G \end{pmatrix} \qquad (92)$$

In the above matrix,

$$G = \begin{pmatrix} -g+k & -\frac{\sqrt{kl}}{2} \\ -\frac{\sqrt{kl}}{2} & g+l \end{pmatrix} \qquad (93)$$

where,

$$g = \frac{1+\frac{C}{r^5}}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}$$

$$k = \frac{\frac{C}{r^5}\cosh^2\alpha\sin^2\phi}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}$$

$$l = \frac{\frac{C}{r^5}\sinh^2\alpha}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}$$

Using $\mathcal{E}\eta\mathcal{E}^{\mathcal{T}} = \mathcal{G}$, we get the vielbien as;

$$\mathcal{E} = \begin{pmatrix} \sqrt{g} & 0 \\ 0 & \hat{E} \end{pmatrix} \tag{94}$$

and $\hat{E}$ is given by;

$$\hat{E} = \begin{pmatrix} \sqrt{g-k} & 0 \\ \frac{1}{2}\sqrt{\frac{kl}{g-k}} & \sqrt{\frac{kl}{4(g-k)}+g+l} \end{pmatrix} \tag{95}$$

The supersymmetry variation of dilatino and gravitini fields of type IIB supergravity in ten dimensions , in string frame is given by []:

$$\delta\lambda_{\pm} = \frac{1}{2}(\Gamma^{\mu}\partial_{\mu}\phi \mp \frac{1}{12}\Gamma^{\mu\nu\rho}H_{\mu\nu\rho})\epsilon_{\pm} + \frac{1}{2}e^{\phi}(\pm\Gamma^{M}F^{(1)}_{M} + \frac{1}{12}\Gamma^{\mu\nu\rho}F^{(3)}_{\mu\nu\rho})\epsilon_{\mp} \tag{96}$$

$$\begin{aligned} \delta\psi^{\pm}_{\mu} = &\left[\partial_{\mu} + \frac{1}{4}(\omega_{\mu\hat{a}\hat{b}} \mp \frac{1}{2}H_{\mu\hat{a}\hat{b}})\Gamma^{\hat{a}\hat{b}}\right]\epsilon_{\pm} + \\ &\frac{1}{8}e^{\phi}\left[\mp\Gamma^{\lambda}F^{(1)}_{\lambda} - \frac{1}{3!}\Gamma^{\lambda\nu\rho}F^{(3)}_{\lambda\nu\rho} \mp \frac{1}{2.5!}\Gamma^{\lambda\nu\rho\alpha\beta}F^{(5)}_{\lambda\nu\rho\alpha\beta}\right]\Gamma_{\mu}\epsilon_{\mp} \end{aligned} \tag{97}$$

where, $H_{\mu\nu\rho}$ is the field strength associated with the NS-NS $B_{\mu\nu}$, $F_{\mu\nu\rho}$ and $F_{\lambda\nu\rho\alpha\beta}$ are the strengths associated with 2-form and 4-form A's respectively.

$$\begin{aligned} F_{\mu\nu\rho} &= \partial_{[\mu}A_{\nu\rho]} \\ H_{\mu\nu\rho} &= \partial_{[\mu}B_{\nu\rho]} \\ F_{\lambda\nu\rho\alpha\beta} &= \partial_{[\lambda}A_{\nu\rho\alpha\beta]} \end{aligned} \tag{98}$$

$$F_{7tr} = \partial_r \left[\frac{\frac{C}{r^4} \sin\phi \cosh\alpha}{1+\frac{C}{r^4}} \right] \tag{99}$$
$$= \frac{\partial_r \left(\frac{C}{r^4}\right) \sin\phi \cosh\alpha}{1+\frac{C}{r^4}}$$

$$F_{78r} = \partial_r \left[\frac{\frac{C}{r^4} \sinh\alpha}{1+\frac{C}{r^4}} \right] \tag{100}$$
$$= \frac{-\partial_r \left(\frac{C}{r^4}\right) \sinh\alpha}{1+\frac{C}{r^4}}$$

$$H_{9tr} = \partial_r \left[-\frac{\frac{C}{r^4} \sinh\alpha \cosh\alpha \cos\phi}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi} \right] \tag{101}$$
$$= -\frac{\partial_r (\frac{C}{r^4} \sinh\alpha \cosh\alpha \cos\phi)}{(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)^2}$$

$$H_{98r} = \partial_r \left[\frac{\frac{C}{r^4} \sin\phi \cos\phi \cosh^2\alpha}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi} \right] \tag{102}$$
$$= \frac{\partial_r (\frac{C}{r^4} \sin\phi \cos\phi \cosh^2\alpha)}{(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)^2}$$

Using the dilatino variation equation we calculate the supersymmetry condition for the string

$$-\Gamma^r\epsilon_\pm \pm \frac{1+\frac{C}{r^4}}{2(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)(1-\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}(\Gamma^{r9t}\sinh\alpha\cosh\alpha\cos\phi$$
$$+\Gamma^{r98}\sin\phi\cos\phi\cosh^2\alpha)\epsilon_\pm + \frac{1}{2(1-\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}(\Gamma^{r7t}\sin\phi\cosh\alpha -$$
$$\Gamma^{r78}\sinh\alpha)\epsilon_\mp = 0 \tag{103}$$

From the above, we get the following conditions.

$$\Gamma^{r9t}\epsilon_\pm = 0 \tag{104}$$
$$(1-\Gamma^{8t}\sin\phi\coth\alpha)\epsilon_\pm = 0 \tag{105}$$
$$\epsilon_\pm = \frac{\Gamma^7\cosh\alpha}{2(1-\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}\left(\Gamma^t\sin\phi - \Gamma^8\tanh\alpha\right)\epsilon_\mp \tag{106}$$

4 Appendix

4.1 T-duality map from type IIA to IIB theory

$$e^{2\phi_b} = \frac{e^{2\phi_a}}{G_{\tilde{x}\tilde{x}}}$$

$$J_{\tilde{x}\mu} = -\frac{B^{(a)}_{\tilde{x}\mu}}{G_{\tilde{x}\tilde{x}}}$$

$$J_{\tilde{x}\tilde{x}} = \frac{1}{G_{\tilde{x}\tilde{x}}}$$

$$B^{(b)}_{\tilde{x}\mu} = -\frac{G_{\tilde{x}\mu}}{G_{\tilde{x}\tilde{x}}}$$

$$A^{(2)}_{\tilde{x}\mu} = A^{(1)}_{\mu} - \frac{A^{(1)}_{\tilde{x}} G_{\tilde{x}\mu}}{G_{\tilde{x}\tilde{x}}}$$

$$B^{(b)}_{\mu\nu} = B^{(a)}_{\mu\nu} + 2\frac{G_{\tilde{x}\mu} B^{(a)}_{\nu\tilde{x}}}{G_{\tilde{x}\tilde{x}}}$$

$$J_{\mu\nu} = G_{\mu\nu} - \frac{G_{\tilde{x}\mu} G_{\tilde{x}\nu} - B^{(a)}_{\tilde{x}\mu} B^{(a)}_{\tilde{x}\nu}}{G_{\tilde{x}\tilde{x}}}$$

$$A^{(2)}_{\mu\nu} = A^{(3)}_{\mu\nu\tilde{x}} - 2A_{\mu} B^{(a)}_{\nu\tilde{x}} + 2\frac{G_{\tilde{x}\mu} B^{(a)}_{\nu\tilde{x}} A^{(1)}_{\tilde{x}}}{G_{\tilde{x}\tilde{x}}}$$

$$A^{(4)}_{\mu\nu\rho\tilde{x}} = A^{(3)}_{\mu\nu\rho} - \frac{3}{2}\left(A_{\mu} B^{(a)}_{\nu\rho} - \frac{G_{\tilde{x}\mu} B^{(a)}_{\nu\rho} A^{(1)}_{\tilde{x}}}{G_{\tilde{x}\tilde{x}}} + \frac{G_{\tilde{x}\mu} A^{(3)}_{\nu\rho\tilde{x}}}{G_{\tilde{x}\tilde{x}}} \right)$$

where $\tilde{x}$ is the Killing coordinate which is T-dualized and $\mu, \nu, \rho \neq \tilde{x}$. [4]

4.2 T-duality map from type IIB to IIA theory

$$e^{2\phi_a} = \frac{e^{2\phi_b}}{J_{\tilde{x}\tilde{x}}}$$

$$G_{\tilde{x}\tilde{x}} = \frac{1}{J_{\tilde{x}\tilde{x}}}$$

$$B^{(a)}_{\tilde{x}\mu} = -\frac{J_{\tilde{x}\mu}}{J_{\tilde{x}\tilde{x}}}$$

$$G_{\tilde{x}\mu} = -\frac{B^{(b)}_{\tilde{x}\mu}}{J_{\tilde{x}\tilde{x}}}$$

$$A^{(1)}_{\mu} = A^{(2)}_{\tilde{x}\mu} + \chi B^{(b)}_{\tilde{x}\mu}$$

$$A^{(3)}_{\tilde{x}\mu\nu} = A^{(2)}_{\mu\nu} + 2\frac{A^{(2)}_{\tilde{x}\mu} J_{\nu\tilde{x}}}{J_{\tilde{x}\tilde{x}}}$$

$$B^{(a)}_{\mu\nu} = B^{(b)}_{\mu\nu} + 2\frac{B^{(b)}_{\tilde{x}\mu} J_{\nu\tilde{x}}}{J_{\tilde{x}\tilde{x}}}$$

$$G_{\mu\nu} = J_{\mu\nu} - \frac{J_{\tilde{x}\mu} J_{\tilde{x}\nu} - B^{(b)}_{\tilde{x}\mu} B^{(b)}_{\tilde{x}\nu}}{J_{\tilde{x}\tilde{x}}}$$

$$A^{(3)}_{\mu\nu\rho} = A^{(4)}_{\mu\nu\rho\tilde{x}} + \frac{3}{2}\left(A^{(2)}_{\tilde{x}\mu} B^{(b)}_{\nu\rho} - B^{(b)}_{\tilde{x}\mu} A^{(2)}_{\nu\rho} - 4\frac{B^{(b)}_{\tilde{x}\mu} A^{(2)}_{\tilde{x}\nu} J_{\rho\tilde{x}}}{J_{\tilde{x}\tilde{x}}} \right)$$

where $\tilde{x}$ is the Killing coordinate which is T-dualized and $\mu, \nu, \rho \neq \tilde{x}$. [4]

4.3 Kaluza Klein compactification mechanism

$$G_{ab} = G^{(10)}_{[a+(D-1),b+(D-1)]}$$

$$B_{ab} = B^{(10)}_{[a+(D-1),b+(D-1)]}$$

$$A^{(a)}_{\mu} = \frac{1}{2} G^{ab} G^{(10)}_{[b+(D-1),\mu]}$$

$$A^{(a+(10-D))}_{\mu} = \frac{1}{2} B^{(10)}_{[a+(D-1),\mu]} - B_{ab} A^{(b)}_{\mu}$$

$$G_{\mu\nu} = G^{(10)}_{\mu\nu} - G^{(10)}_{[a+(D-1),\mu]} G^{(10)}_{[b+(D-1),\nu]} G^{ab}$$

$$B_{\mu\nu} = B^{(10)}_{\mu\nu} - 4 B_{ab} A^{(a)}_{\mu} A^{(b)}_{\nu} - 2(A^{(a)}_{\mu} A^{(a+(10-D))}_{\nu} - A^{(a)}_{\nu} A^{(a+(10-D))}_{\mu})$$

$$\Phi = \Phi^{(10)} - \frac{1}{2} \ln \det G$$

where , $1 \leq a, b \leq 10 - D$ and $0 \leq \mu, \nu \leq (D-1)$ [1]

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